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(54) Title: METHOD AND SYSTEM FOR MODELING GEOLOGICAL STRUCTURES USING AN UNSTRUCTURED FOUR-DIMENSIONAL MESH

(57) Abstract: A method for solving space-time problems involving three-dimensional space wherein an unstructured four-dimensional finite element model is generated and solved to produce a four-dimensional solution. The four-dimensional mesh is generated from a three-dimensional mesh by extruding each of the simplices of the three-dimensional mesh in a time dimension. The four-dimensional prisms formed by extrusion of the three-dimensional simplices are divided into a plurality of four-dimensional simplices which form a four-dimensional finite element model. The elements of the four-dimensional model can be selectively refined to obtain a finer mesh in areas of greater interest, and a coarser mesh in areas which are of less interest. The mesh can be refined in the spatial dimensions and also in the temporal dimension.

METHOD AND SYSTEM FOR MODELING GEOLOGICAL STRUCTURES USING AN UNSTRUCTURED FOUR-DIMENSIONAL MESH

RELATED APPLICATIONS

This application claims priority under 35 U.S.C. § 119(e) to United States Patent Application No. 60/215,697 entitled "Method and System for Oil Reservoir Simulation and

5 Modeling" by Stephen R. Kennon, Kok Thye Lim, Scott A. Canaan, Steven B. Ward, Stuart W. Pond, Jr. and Edward J. Barragy, filed June 29, 2000, which is incorporated by reference as if set forth in its entirety herein.

10 TECHNICAL FIELD OF THE INVENTION

This invention relates generally to methods and systems for modeling physical systems using the finite element method and, more specifically, to methods and systems for creating finite element models for simulating underground reservoirs

15 and corresponding fluid flow, especially in hydrocarbon reservoirs.

BACKGROUND OF THE INVENTION

Physical systems can be modeled mathematically to simulate

20 their behavior. There are a wide variety of means to model these systems, ranging from the very simplistic to the extremely complicated. One of the more complicated means to model physical systems is through the use of the finite element method (FEM). As the name implies, the finite

25 element method involves the representation of individual, finite elements of the physical system in a mathematical model and the solution of this model in the presence of a predetermined set of boundary conditions.

In finite element modeling, the region that is to be
analyzed is broken up into sub-regions called elements.
This process of dividing the region into sub-regions may be
referred to as discretization or mesh generation. The
5   region is represented by functions defined over each
element. This generates a number of local functions that
are much simpler than those which would be required to
represent the entire region. The next step is to analyze
the response for each element. This is accomplished by
10   building an element matrix and vector for each element in
the system. The element matrix defines the properties of
the element within the region, while its vector defines the
forces acting on the element. Once all the element matrices
and vectors have been created, they are combined into a
15   structure matrix equation. This equation relates nodal
responses for the entire structure to nodal forces. Nodes
are specific locations in the model which are used as the
corners or vertices of the elements. After applying
boundary conditions which are constraints on the solution
20   by predetermined properties of the models, the structure
matrix equation can be solved to obtain unknown nodal
responses. Intra-element responses can be interpolated from
nodal values using the functions which were defined over
each element.

25   As indicated above, finite element modeling involves the
creation of a mesh of finite elements. The elements are
defined by nodes within the problem space, which may be 1-,
2-, 3- or higher dimensioned. The nodes are simply points
in space. The lines between the nodes are referred to as
30   "edges." The mesh is typically a structured mesh. In
other words, for a mesh defined in three dimensions, the

elements within the problem space are hexahedrons. For example, they may be cubes or rectangular prisms. (Equivalently, in two dimensional problems, the elements would be rectangles - see FIGURE 1 for an illustration of these elements.) The edges of the hexahedrons are coincident with the edges between the nodes of the mesh. In a simple model, the nodes of the mesh may be regularly spaced to define cubic elements. It is not necessary, however, for the nodes to be evenly spaced in all finite element models. A variety of different sizes and shapes of hexahedral elements can be defined within a single mesh. Smaller elements are typically used to more accurately capture physical phenomena in a local region or to better model the physical shape.

One of the problems with meshes which use hexahedral elements is that they do not lend themselves well to complex geometries which may need to be modeled. In other words, it may be difficult to place nodes sufficiently close to irregular surfaces to accurately determine the solution to the problem along those surfaces. Viewed another way, it is difficult to fill an irregularly shaped object with regularly shaped boxes. This situation is further complicated by limitations on the minimum distance between nodes in a structured mesh. Even when additional nodes (and thus elements) can be defined in the mesh to improve the accuracy of the model, the inclusion of these additional elements causes the resources needed to solve the resulting model to rapidly increase.

These problems are compounded with each additional dimension of the problem. Because of the computational resources required in prior art structured models,

inclusion of a time dimension into a single, structured four-dimensional model is impractical. This is true because meshes made up of hexahedral elements cannot be easily locally refined in size. Consequently, when a

5   solution to a structured three-dimensional model over time is desired, the model must be solved at a first time, then again at a second time, and so on, until a series of time steps covering the desired range is completed. If it is desired to model a temporal feature more accurately, this

10   must be achieved by decreasing the size of the time steps and computing the entire three-dimensional physical model for a greater number of time values. The computing resources which are required to generate the solution obviously increases with the smaller time steps. There is

15   no way to localize the smaller time steps to a particular physical volume of interest.

SUMMARY OF THE INVENTION

One or more of the problems outlined above may be solved by the various embodiments of the invention. Broadly

20   speaking, the invention comprises a method for solving space-time problems involving three-dimensional space by constructing and solving an unstructured four-dimensional finite element model. This method may provide a number of advantages over prior art methods, including "free" mesh

25   generation, faster and easier generation of solutions for the space-time model, elimination of the need for a four-dimensional mesher, and accurate modeling of irregular spatial and temporal features.

In one embodiment, the method comprises providing a three-dimensional mesh which models the system to be solved and extruding the three-dimensional mesh to form a four-dimensional mesh. Both the three-dimensional mesh and the
5   resulting four-dimensional mesh are unstructured. That is, they are not constrained to use hexahedral elements and are not subject to the limitations associated with the use of these elements. The three-dimensional and four-dimensional meshes in the present method comprise elements which are
10 simplices in three-dimensional and four-dimensional space, respectively. The use of simplex elements allows the models to more accurately match the spatial and temporal shape of the modeled system than is possible using a structured mesh of hexahedral elements.

15 The four-dimensional mesh is generated from a three-dimensional mesh by extruding each of the simplices of the three-dimensional mesh in a fourth dimension (time.) The extrusion creates a four-dimensional prism from each of the three-dimensional simplices. These four-dimensional prisms
20 are then divided into several four-dimensional simplices. This process is repeated until the newly generated four-dimensional simplices cover the desired time range. The newly generated four-dimensional simplices form a four-dimensional finite element model. The elements of this
25 model can be selectively adjusted to obtain a finer mesh in areas of greater interest, and a coarser mesh in areas which are of less interest. The mesh can be refined in the spatial dimensions and, unlike time-stepped models, it can also be easily refined in the temporal dimension. The
30 refined four-dimensional mesh may therefore be substantially simplified (i.e., unneeded elements can be

eliminated or collapsed into other elements) so that fewer
computing resources are required to generate a solution for
the model.   (It should be noted that the three-dimensional
mesh can be generated from a two-dimensional mesh in the

5   same way the four-dimensional mesh is extruded from the
three-dimensional mesh.)

In one embodiment, the extrusion process proceeds as
follows.   (Because the process is useful to extrude from
two dimensions to three, as well as from three dimensions

10   to four, the description will be generalized to extrusion
from n dimensions to n+1.)   First, the original, n-
dimensional mesh is duplicated and the duplicate is
displaced in the n+1 dimension.   Each node is connected to
its duplicate by an edge.   The combination of the existing

15   edges and the newly formed edges define an n+1-dimensional
prism corresponding to each of the original simplices.
Then, for each prism, the node number for each of the nodes
in the original n-dimensional simplex is identified.   Based
upon the ordering of the nodes (i.e., whether the first

20   node is numerically higher or lower than the second node,)
a bit pattern is generated.   This bit pattern is used as an
index to a lookup table that contains the combinations of
the nodes that form the vertices of each of the new n+1-
dimensional simplices.

25   **BRIEF DESCRIPTION OF THE DRAWINGS**

Other objects and advantages of the invention may become
apparent upon reading the following detailed description
and upon reference to the accompanying drawings in which:

FIGURE 1 is an illustration of the rectangular and
hexahedral elements which comprise structured finite
element meshes in the prior art;

FIGURE 2 is an illustration of the triangular and
5   tetrahedral elements which comprise unstructured finite
element meshes in the present method;

FIGURES 3a-3c are illustrations of a two-dimensional
spatial curve and both structured and unstructured meshes
that could be used to model the curve;

10   FIGURE 4 is a flow diagram illustrating for generating a
four-dimensional mesh in one embodiment;

FIGURE 5 is a flow diagram illustrating the generation of
an n+1-dimensional mesh from an n-dimensional mesh in one
embodiment;

15   FIGURES 6a - 6d are diagrams illustrating the extrusion of
a one-dimensional simplex into a pair of two-dimensional
simplices in one embodiment;

FIGURE 7 is a flow diagram illustrating the methodology
employed in the creation of the simplices shown in FIGURE
20   6d;

FIGURES 8a-8b are illustrations of the conversion of a one-
dimensional mesh to a two-dimensional mesh in one
embodiment; and

FIGURE 9 is an illustration of the conversion of a two-
25   dimensional simplex into three three-dimensional simplices
in one embodiment.

While the invention is subject to various modifications and alternative forms, specific embodiments thereof are shown by way of example in the drawings and the accompanying detailed description. It should be understood, however,

5 that the drawings and detailed description are not intended to limit the invention to the particular embodiment which is described. This disclosure is instead intended to cover all modifications, equivalents and alternatives falling within the spirit and scope of the present invention as

10 defined by the appended claims.

DETAILED DESCRIPTION OF A PREFERRED EMBODIMENT

The present method and system are directed to solving space-time problems involving three-dimensional space. In the prior art, three-dimensional finite element models were

15 solved for a particular point in time. Put another way, the models were three-dimensional instead of four-dimensional. If a solution for the three-dimensional model over time was required, the three-dimensional model was solved for a first time value, then for a second time

20 value, then a third, and so on. Thus, the finite element model was not a continuous four-dimensional model, but was instead a three-dimensional model which was stepped over a given range of time values. Then, the series of three-dimensional solutions to the three-dimensional model could

25 be combined to construct a four-dimensional solution.

The present method and system employ a four-dimensional finite element model to generate a four-dimensional solution. More particularly, an unstructured finite element mesh is generated to model the desired physical

30 system over a desired time span. This four-dimensional

model is then solved to generate a four-dimensional
solution (a solution for the three-dimensional system over
time.) The unstructured four-dimensional finite element
model is capable of following irregular spatial and
temporal features of the physical system which is being
modeled with much greater accuracy than can be achieved
using a regular (hexahedral) mesh. The four-dimensional
mesh is generated from a three-dimensional mesh by
extruding each of the simplices of the three-dimensional
mesh in a fourth dimension (time.) The extrusion creates a
four-dimensional prism from each of the three-dimensional
simplices. These four-dimensional prisms are then divided
into several four-dimensional simplices which form a four-
dimensional finite element model. The elements of this
model can be selectively refined to obtain a finer mesh in
areas of greater interest, and a coarser mesh in areas
which are of less interest. The mesh can be refined in the
spatial dimensions and, unlike time-stepped models, it can
also be easily refined in the temporal dimension.

It will be helpful to first define a few terms.

A node is a point in space. In finite element modeling,
nodes form the vertices of the elements which are modeled.
The nodes also form part of a mesh, where nodes and edges
define the boundaries between elements in the modeled
space.

An edge is a line between two nodes which form vertices of
an element. The edges form part of the mesh which defines
the boundaries between elements in the modeled space.

An element is a spatial configuration that encloses space
in a given dimension. It is bounded by nodes and edges.

A simplex is a spatial configuration of n dimensions determined by n + 1 points in a space of dimension equal to or greater than n. In other words, a simplex is a geometric spatial element having the minimum number of

5 boundary points necessary to enclose a space in a given number of dimensions. For example, in two dimensions, a simplex comprises a triangle, together with the interior area bounded by the triangle (see FIGURE 2.) Two points are insufficient to form a simplex in two-dimensional space

10 because no area is bounded by the points (and the lines which interconnect them.) While four points may be sufficient to bound a two-dimensional area, they do not comprise the minimum number of boundary points by which the two-dimensional area can be bounded. In three dimensions,

15 a simplex comprises a tetrahedron, which is bounded by four vertices (see FIGURE 2.) In four dimensions, a simplex comprises a hypertetrahedron (sometimes referred to as a hypertet) having five vertices.

A mesh is a collection of elements that fill a space.

20 These elements are representative of a system which resides in that space. Because each element can be defined by a plurality of nodes and/or the edges between those nodes, a mesh may alternatively be considered a collection of nodes and/or the edges between them. At various points in this

25 disclosure, "mesh" will be used to alternately refer to collections of elements or nodes/edges, depending upon the context in which the term is used. The mesh may also be referred to herein as a finite element model or simply a model.

30 In the present system and method, a four-dimensional mesh representing a volume to be modeled and a temporal range

over which the volume is to be modeled is constructed. The

mesh is an unstructured mesh of four-dimensional simplex

elements. That is, each element in the mesh is defined by

five nodes which form its vertices. The elements are not

5   constrained by the same requirements as structured meshes

(e.g., minimum size of elements) and are therefore capable

of more closely matching nodes of the elements to points of

interest (e.g., object surfaces) and more accurately

modeling physical systems.

10   An example is illustrated in FIGURES 3a - 3c. FIGURE 3a

shows a curve which represents a surface in two-dimensional

space. FIGURE 3b shows a structured two-dimensional mesh

which can be constructed to model the surface shown in

FIGURE 3a. The curve which is to be modeled is also shown

15   in this figure. It can be seen that the elements of the

structured mesh are larger in the portions of the mesh

which do not touch the curve. Some of the elements in the

mesh which are closer to the curve have been broken down

into smaller elements. The elements are generally broken

20   down in this manner in order to more accurately model

regions around the curve. The accuracy of this

representation of the curve, however, is limited because

the minimum size of the elements may be limited. Further,

regardless of the minimum size of the elements, the

25   elements are all rectangular and consequently cannot follow

the varying slope of the curve.

FIGURE 3c shows an unstructured two-dimensional mesh that

can be constructed to model the curve. It can be seen that

this mesh uses two-dimensional simplices (i.e., triangles)

30   rather than rectangular elements. While the mesh shown in

this figure also begins with larger elements (i.e.,

elements whose size is roughly equivalent to the elements
of the structured mesh of FIGURE 3b,) the elements are not
simply subdivided until their nodes become arbitrarily
close to the curve, as was the case with the structured

5   mesh.   Instead, the nodes of the elements can be shifted
(in addition to subdividing the elements, if necessary) so
that the nodes are arbitrarily close to the curve.
Further, by shifting the nodes to fall on the curve, the
edges of the simplices can follow the curve and model it

10   with much more accuracy than the structured mesh (note the
heavy line in each of FIGURES 3b and 3c which shows the
edges that follow the modeled curve.)

FIGURES 3b and 3c illustrate the improved accuracy with
which the unstructured mesh can follow the physical

15   geometry of the system being modeled.   It should be noted
that this same advantage is also achieved, and may be even
more important in the temporal dimension, since it can
effectively enable variable time steps.   As shown above,
both structured and unstructured meshes can, to some

20   extent, resolve smaller spatial features by using a finer
mesh in a region of interest.   That is, for a selected
region of XYZ space, a finer mesh can be used.   The
unstructured mesh can, however, do essentially the same
thing in the temporal dimension, while the structured mesh

25   cannot, when used in the context of prior art time-stepping
schemes as described above.

As indicated above, prior art methods do not actually solve
a four dimensional model.   Instead, a three-dimensional
model is solved at each of a collection of time steps.

30   Thus, if increased accuracy is desired in a particular time
interval, the entire three-dimensional spatial model must

be solved for an increased number of time values (using
smaller time steps.) The structured mesh cannot be refined
to have a first time step in a in a first spatial region
and at the same time have a different time step in a second

5   spatial region. Using the unstructured mesh of the present
system and method, however, this is easily achieved.

The usefulness of this capability is illustrated by the
following example. If a disturbance such as a pressure
wave propagates through a reservoir, it may be desirable to

10   increase the number of elements in the vicinity of the
disturbance. Because the disturbance propagates through
the reservoir, the location at which the fine mesh is
needed changes with time. Presumably, the disturbance will
propagate through the entire space which is being modeled,

15   so, at one time or another, each location in the modeled
space will need to have an increased number of elements. A
structured three-dimensional mesh does not change with
time. It is simply time-stepped, with each element in the
mesh being advanced by the same interval (size of time

20   step.) Thus, each region in the modeled space must have
the maximum number of elements that will be desired at any
time. A solution for the model at any given time requires
solutions for all of these elements, regardless of whether
the additional elements are needed at that time. As a

25   result of the increased number of elements in the mesh,
increased computing resources are required to generate a
solution.

The present system and method solves this problem by
generating a four-dimensional mesh that can be refined for

30   particular regions of both space and time. Just as the two
dimensional mesh illustrated in FIGURE 3c can be refined to

include additional elements in particular areas of X-Y

space, the present system and method enable refinement of

the four-dimensional mesh to add elements (or even collapse

elements) in particular regions of X-Y-Z-time space.

5 Because the mesh can be refined in time as well as space,

the mesh need not include in a spatial region the maximum

number of elements that will be needed at any time. It can

instead include only those elements that will be needed at

the appropriate time and space, effectively varying the

10 time step from one spatial region to another. As a result,

there are less elements in the mesh corresponding to any

given time, and less computing resources are needed to

compute a solution for the modeled system.

The generation of the four-dimensional mesh can be

15 performed in a number of ways. A description of a

preferred method follows. Referring to FIGURE 4, a flow

diagram illustrating the method is shown. First, an

unstructured three-dimensional mesh which is representative

of the physical system to be modeled (i.e., a mesh in X-Y-Z

20 space) is generated. This three-dimensional mesh may be

generated by any suitable means. This three-dimensional

mesh is then extruded in the temporal dimension. (A

preferred method of extrusion of the three-dimensional mesh

will be described in more detail below.) The three-

25 dimensional mesh is extruded over a certain interval which

is equivalent to the minimum time step that would be

desired at any region in the three-dimensional mesh during

that interval. The minimum time step is driven by both the

minimum mesh size and the largest velocity values in the

30 system. The extrusion creates a plurality of four-

dimensional prisms, each of which is then broken up into

three four-dimensional simplices, creating an unstructured
four-dimensional mesh extending over the interval. The
extrusion and creation of simplices is then repeated for
each succeeding interval over the desired time range for

5   the four-dimensional model. (It should be noted that each
successive extrusion is made from the three-dimensional
mesh displaced in the previous extrusion, and not from the
four-dimensional simplices.) The four-dimensional mesh is
then examined to identify regions in which the minimum time

10   step is not necessary (for example, after a disturbance has
passed through a region.) Selected nodes in these regions
are eliminated, and the elements associated with the
eliminated nodes are collapsed into fewer elements. This
results in an unstructured four-dimensional mesh which has

15   varying time steps in different regions of the modeled
four-dimensional space. (It should be noted that it is
also possible to generate a mesh that uses larger time
steps and then add nodes in regions of interest, wherein
the new nodes are at times which are intermediate to the

20   original time steps.)

As indicated above, the three-dimensional mesh which models
the physical region to be modeled can be generated in
various ways. In one embodiment, the three-dimensional
mesh is generated by extruding a two dimensional (e.g., X-

25   Y) mesh in a third dimension (Z). The generation of the
three-dimensional mesh from the two dimensional mesh is
performed in essentially the same manner as the generation
of the four-dimensional mesh from the three-dimensional
mesh. This method can be generalized to any number of

30   dimensions. Referring to FIGURE 5, a flow diagram
illustrating one embodiment of this method is shown.

According to this embodiment, an n-dimensional mesh is
provided. This mesh is then extruded into another
dimension (the n+1th dimension) to form n+1-dimensional
prisms. These n+1-dimensional prisms are then divided into
5 n+1-dimensional simplices, thereby forming an n+1-
dimensional mesh. The n+1-dimensional mesh which is
created in this manner can then be refined if necessary to
add one or more nodes in areas of interest or to eliminate
nodes where they are not needed.

10 The general method of FIGURE 5 can be used to extrude a
mesh of a given dimension into an additional dimension.
For example, a one-dimensional (e.g., X) mesh can be used
to form a two-dimensional (e.g., X-Y) mesh, or a mesh in
two dimensions (e.g., X and Y) can be extruded into a third
15 dimension (e.g., Z or time.) Thus, a finite element model
in two-dimensional space can be used to form in a model in
three-dimensional space, or in two-dimensional space-time.
In the same manner, a finite element model in three-
dimensional space can be extruded to generate a three-
20 dimensional space-time model.

Referring to FIGURES 6a - 6d, a series of drawings
illustrating the extrusion of a one-dimensional simplex
into a pair of two-dimensional simplices is shown. In
FIGURE 6a, a one-dimensional simplex 11 comprising nodes 0
25 and 1 is shown on the X-axis. The line between these nodes
comprises the only edge of the simplex.

In FIGURE 6b, the one-dimensional simplex 11 is extruded in
the Y direction. That is, a copy of the simplex is created
and is displaced from the original simplex in the Y
30 direction. This copy is indicated by the reference numeral

12. The new one-dimensional simplex 12 comprises nodes 2
and 3 into a single edge extending between them. The two
one-dimensional simplices are connected by a pair of edges
13, 14 extending from nodes 0 and 1 to nodes 2 and 3,
5   respectively. Thus, a two-dimensional prism 15 is created
by the extrusion of the original one-dimensional simplex 11
in an additional dimension.

Because the two-dimensional prism which is created by the
extrusion of the original one-dimensional simplex 11 is not
10 itself a simplex in two dimensions, it is necessary to
divide the prism 15 into two simplices. This is
illustrated in FIGURE 6c. In this figure, a new edge 16
extending from the node 0 to node 3 is shown. This edge 16
divides the two-dimensional prism 15 into a pair of two-
15 dimensional simplices. These simplices are indicated by
the reference numerals 17 and 18 in FIGURE 6d.

It should be noted that two-dimensional prism 15 can be
divided into two-dimensional simplices in either of two
ways: by dividing the prism between nodes 0 and 3, as
20 shown in FIGURE 6c; or by dividing the prism between nodes
1 and 2. In the first case, two-dimensional simplices 17
and 18 are created. In the second case, the two-
dimensional simplices having the vertices (0, 1, 2) and (1,
2, 3) are created. Both sets of simplices are equally
25 valid. The first set of simplices, however, was chosen in
this example because of the particular methodology that was
employed. Using this methodology, edge 16 originates at
the lower-numbered node of the original one-dimensional
simplex and terminates at the opposite node of the two-
30 dimensional prism 15.

The methodology employed in the creation of simplices 17
and 18 is illustrated in the flow diagram of FIGURE 7.
Once a prism has been created by extruding a simplex, the
node number for each of the nodes in the original simplex

5   is identified. Based upon the ordering of the nodes (i.e.,
whether the first node is numerically higher or lower than
the second node,) a bit pattern is generated. This bit
pattern is used as an index to a lookup table that contains
the vertices of the new simplices. The selected vertices

10   are then used to define the new simplices.

It may be useful to consider the example of FIGURE 6 prior
to identification of the node numbers. If it is assumed
that nodes 0-3 are initially known only as nodes A-D, and
it is assumed that node A is extruded to node C and node B

15   is extruded to node D, then the lookup table might look
like Table 1 below. (It should be noted that the first
column of table 1 and does not show the bit pattern itself,
but instead shows the relationship of A and B which is
represented by the bit pattern. In some embodiments, the

20   lookup table contains only the sets of vertices which
define the simplices, and the bit pattern is equivalent to
the line/entry number of the respective set of vertices.)

Table 1

A<B	{A,C,D}, {A,B,D}
A>B	{A,B,C}, {B,C,D}

25   In the example of FIGURE 6, A=0 and B=1, a bit pattern
corresponding to A<B is generated. In one embodiment, this
bit pattern could be 0. This bit pattern is used to index

into Table 1. Accordingly, the first entry in the table
(corresponding to A<B) is selected. The vertices listed in
this entry are {A, C, D} and {A, B, D}. Since A-D are
equal to 0-3, respectively, in the figure, the new
5   simplices are defined by the vertices {0, 2, 3} and {0, 1,
3}.

If the nodes had been examined in a different order, the
same result would be reached. In the example above, node 0
was the "A" node, while node 1 was the "B" node. If this
10   were reversed (i.e., if node 0 was the "B" node, while node
1 was the "A" node,) the same vertices would be selected
for the new simplices. If A=1, B=0, C=3 and D=2, then A
would have been greater than B, and the second entry in the
table (corresponding to A>B) would have been selected. The
15   vertices listed in this entry are {A, B, C} and {B, C, D}.
Substituting the node numbers for A-D, the vertices would
be {1, 0, 3} and {0, 3, 2}, which are the same vertices
which were selected in the first instance.

It can also be seen from this example that, if the node
20   numbers were reversed (i.e., if node 1 were to the left of
node 0 on the X axis,) the new simplices would have been
selected so that edge 16 extended from the lower, right-
hand node to the upper, left-hand node. It should be noted
that, in this instance, the new edge would still extend
25   from node 0 to the opposite node of the prism.

While the example of the FIGURE 6 shows the conversion of
only a single one-dimensional simplex into a pair of two-
dimensional simplices, this method can be used to generate
additional two-dimensional simplices based upon the
30   original one-dimensional simplex, or based upon a mesh of

one-dimensional simplices.   Thus, a one-dimensional mesh as
shown in FIGURE 8a can be used to generate a mesh of two-
dimensional simplices as shown in FIGURE 8b.

The method described above for the generation of two-
5   dimensional simplices from one-dimensional simplices can be
generalized to n dimensions.   That is, it can be used to
generate n+1-dimensional simplices from n-dimensional
simplices.   For example, using this method, a two-
dimensional mesh can be used to generate a three-
10   dimensional mesh, or a three-dimensional mesh can be used
to generate a four-dimensional mesh.   The method is
therefore useful in finite element modeling of three-
dimensional space, two-dimensional space and time, and
three-dimensional space and time.

15   Referring to FIGURE 9 an example of the generation of
three-dimensional simplices from a two-dimensional simplex
is illustrated.   (The generation of four-dimensional
simplices will not be shown here because of the difficulty
of graphically representing four dimensions in the
20   drawings.)

Triangle 20 is a two-dimensional simplex in the X-Y plane.
Triangle 20 is defined by vertices A, B and C.   The first
step in generating three-dimensional simplices from this
two-dimensional simplex is to extrude triangle 20 in the Z
25   direction.   Three-dimensional prism 21 is formed by this
extrusion.   Prism 21 has vertices A, B, C, D, E and F.

The next step in the process is to determine the node
numbers associated with each of the vertices of triangle 20
(and/or prism 21.)   Based upon the node numbers, a bit
30   pattern is generated.   This bit pattern is used to index

into a lookup table which contains the sets of vertices
corresponding to each possible ordering of the node
numbers. It should be noted that the ordering of the node
numbers for nodes D, E and F are assumed to be identical to

5   be ordering of the node numbers for nodes A, B and C.
Because the node numbers of nodes D, E and F are assigned
when prism 21 is extruded from triangle 20, it is a simple
matter to ensure that this is true. The bit pattern which
is generated based upon the node numbers of triangle 20 (or

10   prism 21) is then used to select the proper entry from the
lookup table.

In one embodiment, the lookup table is as shown below in
Table 2.

Table 2

1	A<B<C	{A,B,C,F}, {A,D,E,F}, {B,A,E,F}
2	B<A<C	{A,B,C,F}, {B,A,D,F}, {E,B,D,F}
3	unused	{N,N,N,N}, {N,N,N,N}, {N,N,N,N}
4	B<C<A	{B,C,A,D}, {B,E,F,D}, {C,B,F,D}
5	A<C<B	{A,C,F,E}, {C,A,B,E}, {D,A,F,E}
6	unused	{N,N,N,N}, {N,N,N,N}, {N,N,N,N}
7	C<A<B	{A,C,D,E}, {C,A,B,E}, {C,F,D,E}
8	C<B<A	{B,C,A,D}, {C,B,E,D}, {F,C,E,D}

15

Each entry in table 2 contains three sets of vertices.
Each of the sets of vertices defines one of the simplices
into which the prism is divided. There are six possible
orderings of the three vertices of triangle 20. The

possible orderings are: A<B<C; B<C<A; C<A<B; A<C<B; C<B<A;
and B<A<C.   Each of these orderings has a particular bit
pattern associated with it.   In the embodiment which
implements Table 2, the bit pattern is represented in three

5   bits, so there can be eight possible bit patterns.
Consequently, the table contains eight entries
corresponding to the possible bit patterns.   Two of the
entries in table 2 correspond to invalid bit patterns.   If
either of these bit patterns is generated, an error is

10   indicated.

In one embodiment, a bit shifting scheme is used to
generate a bit pattern which is the basis for indexing into
the lookup table.   It is the use of this particular bit
shifting scheme which necessitates the eight-entry table

15   shown above.   It should be noted that other means for
generating an appropriate bit pattern can be used in other
embodiments.   Likewise, it is not necessary to generate a
bit pattern in other embodiments if another means for
indexing into the lookup table is implemented.

20   Referring again to FIGURE 9, if it is assumed that the node
numbers for the vertices follow the same ordering as the
corresponding letters by which they are identified (i.e.,
if A<B<C<D<E<F,) then according to the present method, a
bit pattern corresponding to A<B<C will be generated.   This

25   bit pattern will be used to index into Table 2 and select
the first entry (which corresponds to A<B<C.)   As defined
by this entry, the sets of vertices which define the new
three-dimensional simplices are {A, B, C, F}, {A, D, E, F}
and {B, A, E, F}.

·Prism 21 is shown in the lower, left-hand portion of FIGURE
9 with the edges of the new simplices indicated by the
dashed lines. The separate three-dimensional simplices are
shown on the lower, right hand side of the figure. It can
5   be seen that each one of the new simplices has the four
vertices defined by the corresponding one of the sets of
vertices included in the first entry of Table 2.

Thus, using the present methodology, a two-dimensional mesh
can be converted to a three-dimensional mesh. Just as the
10 present methodology can be extended from the first case
described above (generating a two-dimensional mesh from a
one-dimensional mesh,) to the second case (generating a
three-dimensional mesh from a two-dimensional mesh,) it
can be extended to n dimensions. That is, a mesh of
15 dimension n+1 can be generated from a mesh of dimension n.
While examples of the generation of meshes of greater than
four dimensions will not be given here due to the
difficulty of illustrating four-dimensional figures, the
methodology is applied in the same manner as described
20 above.

It should be noted that the lookup table which is used to
determine the vertices of the simplices in the n+1th
dimension will increase rapidly in size. For example, when
generating a two-dimensional mesh from a one-dimensional
25 mesh, there are only two possible choices when determining
the new, two-dimensional simplices, so only two entries are
necessary in the lookup table. When generating a three-
dimensional mesh from a two-dimensional mesh, however,
there are six possible choices of the new, three-
30 dimensional simplices. In the scheme described above, a
table having eight entries is used. When it is desired to

generate a four-dimensional mesh from a three-dimensional mesh, there are 24 possible choices for the new, four-dimensional simplices. It is evident that the number of choices is (n+1)!

5   It should be noted that, because of the particular bit shifting scheme which is employed in the preferred embodiment, the lookup table which contains the vertices of the new, four-dimensional simplices will have 64 entries. Only 24 of the entries in this table will be valid. These
10   entries will correspond to bit patterns associated with the 24 possible permutations of the order of the nodes of the three-dimensional simplex from which the new, four-dimensional simplices are derived. The remaining entries will correspond to invalid bit patterns. If any of these
15   invalid bit patterns is selected, an error will be indicated.

The use of the present methodology provides for extremely fast generation of meshes having an additional dimension. Because the present methodology extrudes the existing mesh
20   (of dimension n) to form a plurality of prisms (of dimension n+1) and then divides these prisms into n+1-dimensional simplices through the use of a lookup table, it is extremely fast. This methodology eliminates the need for a prior art mesher of the type which has previously
25   been necessary to generate a four-dimensional mesh. Consequently, the cost of purchasing a mesher or expending the time and effort to develop code for a mesher is also eliminated. The present methodology therefore effectively enables "free" four-dimensional mesh generation.

While the present invention has been described with
reference to particular embodiments, it should be
understood that the embodiments are illustrative and that
the scope of the invention is not limited to these

5   embodiments. Many variations, modifications, additions and
improvements to the embodiments described above are
possible. Particularly, these variations may include
computer readable media (such as floppy disks, CD-ROMs,
DVD-ROMs, etc.,) storage devices, computer memories and the

10   like which contain software, firmware or other programming
embodying the foregoing methods. It is contemplated that
these variations, modifications, additions and improvements
fall within the scope of the invention as detailed within
the following claims.

CLAIMS

1.   A method for modeling a three-dimensional system in time comprising:
         roviding a three-dimensional mesh, wherein the three-
5   dimensional mesh corresponds to a system to be modeled;
generating a four-dimensional mesh based from the three-
dimensional mesh, wherein the fourth dimension corresponds
to a time over which the system is to be modeled; and
generating a solution the four-dimensional mesh.
10

2.   The method of claim 1 wherein providing the three-
dimensional mesh comprises providing a three-dimensional
mesh that contains data corresponding to a geological
structure.
15

3.   The method of claim 1 wherein the four-dimensional
mesh comprises a plurality of nodes and wherein the
plurality of nodes have a plurality of different time
values associated therewith.
20

4.   The method of claim 1 wherein the three-dimensional
mesh comprises a plurality of three-dimensional simplices
in a an unstructured mesh.

25   5.   The method of claim 1 wherein the four-dimensional
mesh comprises a plurality of four-dimensional simplices in
a an unstructured mesh.

      6.   The method of claim 1 wherein generating the four-
30   dimensional mesh comprises extruding the three-dimensional
mesh in a fourth dimension.

7.   The method of claim 6 wherein extruding the three-
dimensional mesh in a fourth dimension comprises:

        or each three-dimensional simplex of the three-
5   dimensional mesh,

        reating a four-dimensional prism by extruding the
three-dimensional simplex in the fourth dimension, and

        iving the four-dimensional prism into four four-
dimensional simplices.

10

8.   The method of claim 7 wherein dividing the four-
dimensional prism into four four-dimensional simplexes
comprises identifying the nodes of the four-dimensional
prism and defining subsets of the nodes of the four-
15   dimensional prism as vertices of the four-dimensional
simplices.

9.   The method of claim 8 wherein defining subsets of the
nodes of the four-dimensional prism as four-dimensional
20   simplices comprises identifying a node number associated
with each of the nodes of the four-dimensional prism,
determining a bit pattern based on the node numbers,
indexing into a lookup table based on the bit pattern, and
reading the vertices of the four-dimensional simplices from
25   the table.

10.   The method of claim 1 further comprising collapsing a
plurality of simplices in the four-dimensional mesh into a
single simplex in the four-dimensional mesh.

30

11.   The method of claim 1 further comprising adding one or
more additional nodes to the four-dimensional mesh and

creating a plurality of new simplices which include the one
or more additional nodes.

12.   The method of claim 1 wherein a time step
5   corresponding to a first time represented by the four-
dimensional mesh varies across a three-dimensional spatial
region of the four-dimensional mesh.

13.   The method of claim 1 wherein providing the three-
10   dimensional mesh comprises providing a two dimensional mesh
and extruding the two dimensional mesh into the three-
dimensional mesh.

14.   A computer readable medium having one or more
15   instructions embodied therein, wherein the instructions are
configured to cause a computer to:
    onvert a three-dimensional mesh to a four-dimensional
mesh, wherein the three-dimensional mesh corresponds to a
system to be modeled and the four-dimensional mesh
20   corresponds to the system to be modeled over a range of
time; and
    enerate a solution the four-dimensional mesh.

15.   The computer readable medium of claim 14 wherein the
25   instructions are configured to cause the computer to read
data associated with the three-dimensional mesh.

16.   The computer readable medium of claim 14 wherein the
four-dimensional mesh comprises a plurality of nodes and
30   wherein the instructions are configured to cause the
computer to associate the plurality of nodes with a
plurality of different time values.

17.   The computer readable medium of claim 14 wherein the three-dimensional mesh comprises a plurality of three-dimensional simplices in a an unstructured mesh.

5   18.   The computer readable medium of claim 14 wherein the four-dimensional mesh comprises a plurality of four-dimensional simplices in a an unstructured mesh.

19.   The computer readable medium of claim 14 wherein the
10   instructions are configured to cause the computer to generate the four-dimensional mesh by extruding the three-dimensional mesh in a fourth dimension.

20.   The computer readable medium of claim 6 wherein
15   extruding the three-dimensional mesh in a fourth dimension comprises:
    or each three-dimensional simplex of the three-dimensional mesh,
        creating a four-dimensional prism by extruding the
20   three-dimensional simplex in the fourth dimension, and
        dividing the four-dimensional prism into four four-dimensional simplices.

21.   The computer readable medium of claim 20 wherein
25   dividing the four-dimensional prism into four four-dimensional simplexes comprises identifying the nodes of the four-dimensional prism and defining subsets of the nodes of the four-dimensional prism as vertices of the four-dimensional simplices.

30

22.   The computer readable medium of claim 21 wherein defining subsets of the nodes of the four-dimensional prism

as four-dimensional simplices comprises identifying a node
number associated with each of the nodes of the four-
dimensional prism, determining a bit pattern based on the
node numbers, indexing into a lookup table based on the bit

5   pattern, and reading the vertices of the four-dimensional
simplices from the table.

23. The computer readable medium of claim 14 wherein the
instructions are further configured to cause the computer

10   to collapse a plurality of simplices in the four-
dimensional mesh into a single simplex in the four-
dimensional mesh.

24. The computer readable medium of claim 14 wherein the

15   instructions are further configured to cause the computer
to add one or more additional nodes to the four-dimensional
mesh and create a plurality of new simplices which include
the one or more additional nodes.

20   25. The computer readable medium of claim 14 wherein a
time step corresponding to a first time represented by the
four-dimensional mesh varies across a three-dimensional
spatial region of the four-dimensional mesh.

25   26. The computer readable medium of claim 14 wherein the
instructions are configured to cause the computer to
generate the three-dimensional mesh by extruding a two
dimensional mesh into the three-dimensional mesh.

27. A method for generating a second mesh of dimension n+1 from a first mesh of dimension n, wherein each mesh includes a plurality of simplices, the method comprising:

for each n-simplex of the first mesh,

5   creating an (n+1)-prism by extruding the simplex in the n+1th dimension, and

dividing the (n+1)-prism into n+1 (n+1)-simplices.

28. The method of claim 27 wherein extruding the simplex

10   in the n+1th dimension comprises duplicating the original simplex, displacing the duplicate simplex from the original simplex in the n+1th dimension, and defining an edge between each node of the original simplex and the corresponding node of the duplicate simplex.

15

29. The method of claim 27 wherein dividing the (n+1)-prism into n+1 (n+1)-simplexes comprises identifying the nodes of the (n+1)-prism and defining subsets of the nodes of the (n+1)-prism as vertices of the (n+1)-simplices.

20

30. The method of claim 29 wherein defining subsets of the nodes of the (n+1)-prism as (n+1)-simplices comprises identifying a node number associated with each of the nodes of the (n+1)-prism, determining a bit pattern based on the

25   node numbers, indexing into a lookup table based on the bit pattern, and reading the vertices of the (n+1) simplices from the table.

31. A computer readable medium having one or more

30   instructions embodied therein, wherein the instructions are configured to cause a computer to:

generate a second mesh of dimension n+1 from a first

mesh of dimension n, wherein each mesh includes a plurality

of simplices, and wherein for each n-simplex of the first

mesh, an (n+1)-prism is created by extruding the simplex in

5   the n+1th dimension, and

the (n+1)-prism is divided into n+1 (n+1)-simplices.

32.   The computer readable medium of claim 31 wherein

extruding the simplex in the n+1th dimension comprises

10   duplicating the original simplex, displacing the duplicate

simplex from the original simplex in the n+1th dimension,

and defining an edge between each node of the original

simplex and the corresponding node of the duplicate

simplex.

15

33.   The computer readable medium of claim 31 wherein

dividing the (n+1)-prism into n+1 (n+1)-simplexes comprises

identifying the nodes of the (n+1)-prism and defining

subsets of the nodes of the (n+1)-prism as vertices of the

20   (n+1)-simplices.

34.   The computer readable medium of claim 33 wherein

defining subsets of the nodes of the (n+1)-prism as (n+1)-

simplices comprises identifying a node number associated

25   with each of the nodes of the (n+1)-prism, determining a

bit pattern based on the node numbers, indexing into a

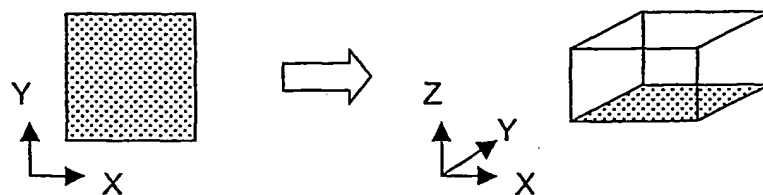lookup table based on the bit pattern, and reading the
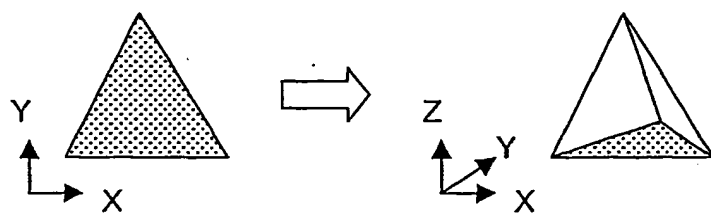
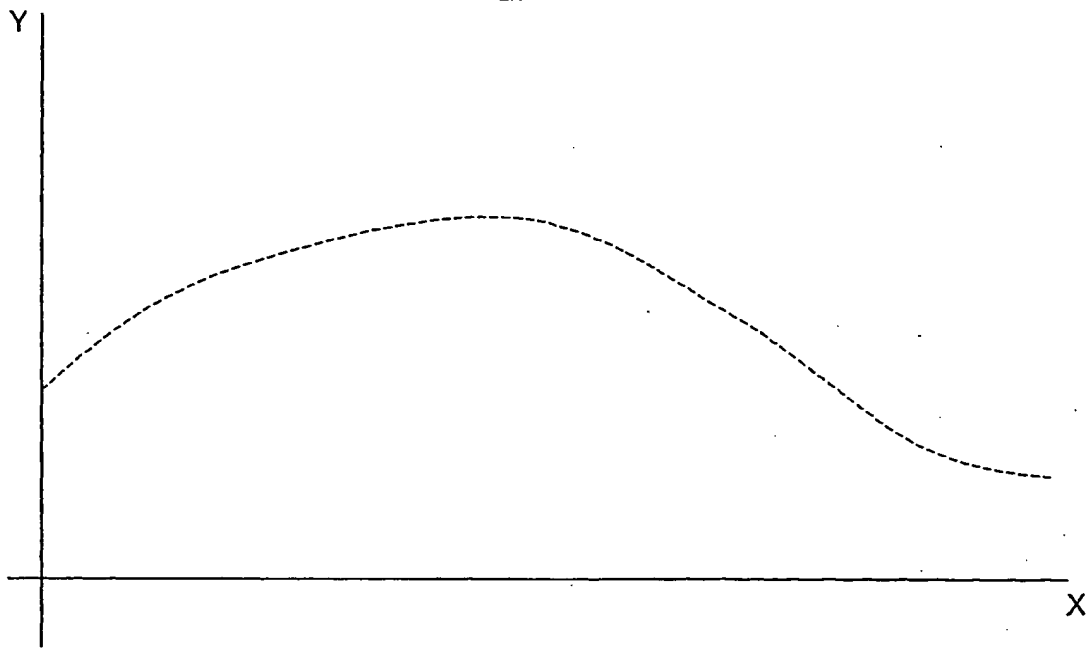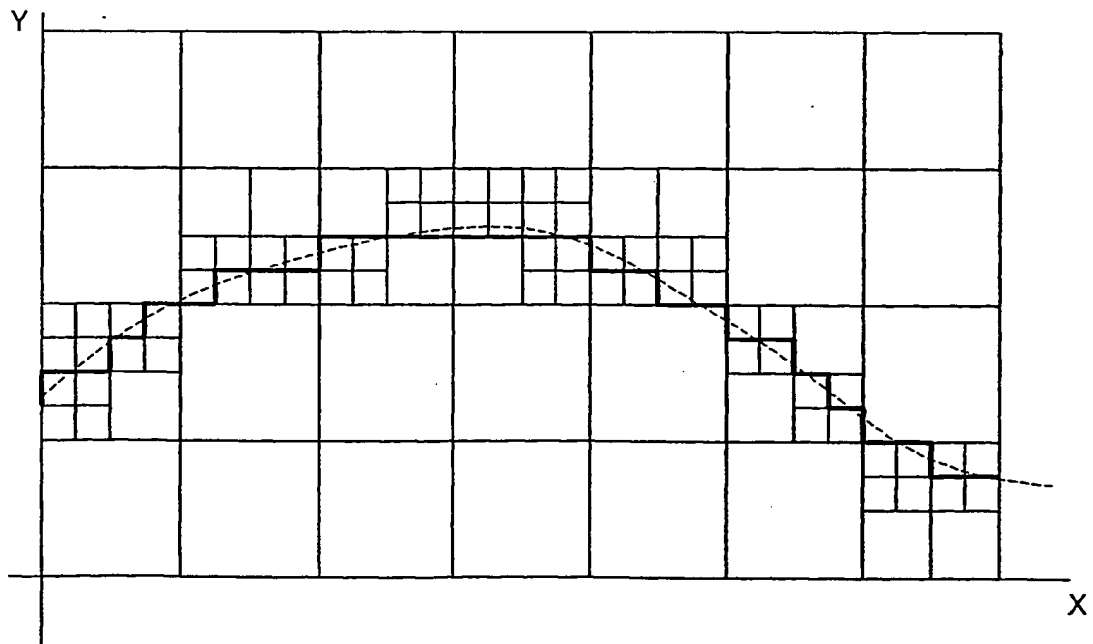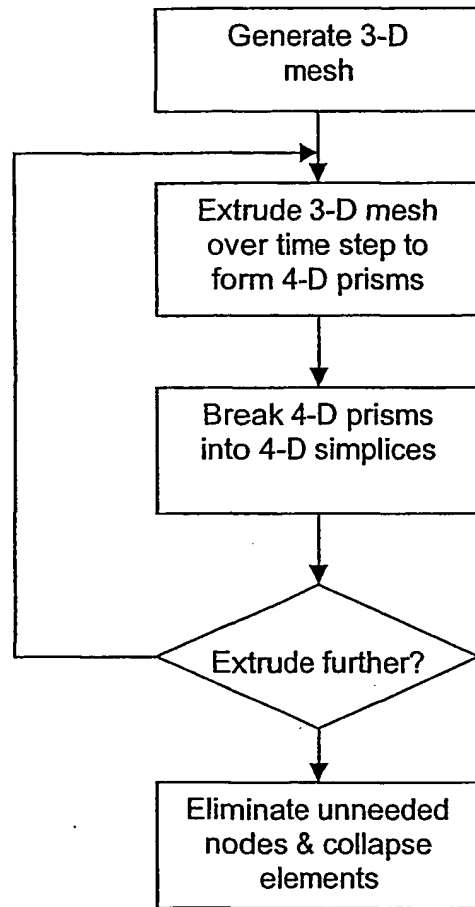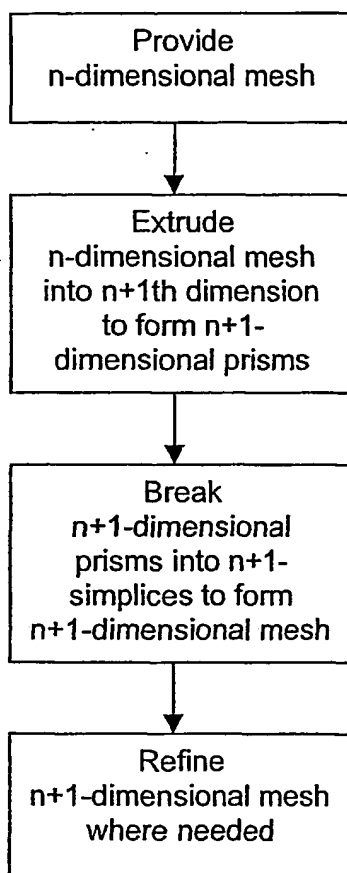vertices of the (n+1) simplices from the table.

Fig. 1

Fig. 2

Fig. 3a

Fig. 3b

Fig. 3c

```
┌─────────────────────┐
│   Generate 3-D      │
│      mesh           │
└─────────────────────┘
          │
          ▼
┌─────────────────────┐
│  Extrude 3-D mesh   │
│  over time step to  │
│  form 4-D prisms    │
└─────────────────────┘
          │
          ▼
┌─────────────────────┐
│   Break 4-D prisms  │
│   into 4-D simplices│
└─────────────────────┘
          │
          ▼
     ◇ Extrude further? ◇
          │
          ▼
┌─────────────────────┐
│ Eliminate unneeded  │
│  nodes & collapse   │
│     elements        │
└─────────────────────┘
```

Fig. 4

```
┌─────────────────────────┐
│        Provide          │
│   n-dimensional mesh     │
└─────────────────────────┘
             │
             ▼
┌─────────────────────────┐
│        Extrude           │
│   n-dimensional mesh     │
│  into n+1th dimension    │
│     to form n+1-         │
│   dimensional prisms     │
└─────────────────────────┘
             │
             ▼
┌─────────────────────────┐
│         Break            │
│    n+1-dimensional       │
│    prisms into n+1-      │
│    simplices to form     │
│  n+1-dimensional mesh    │
└─────────────────────────┘
             │
             ▼
┌─────────────────────────┐
│        Refine            │
│  n+1-dimensional mesh    │
│     where needed         │
└─────────────────────────┘
```
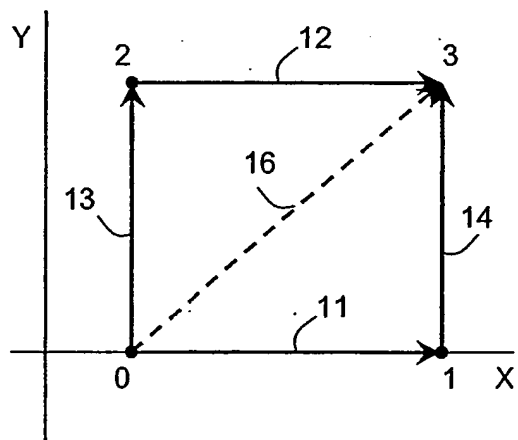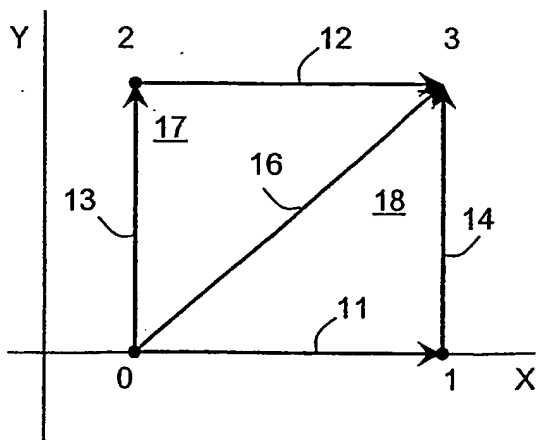
Fig. 5

Fig. 6a

Fig. 6b

Fig. 6c

Fig. 6d
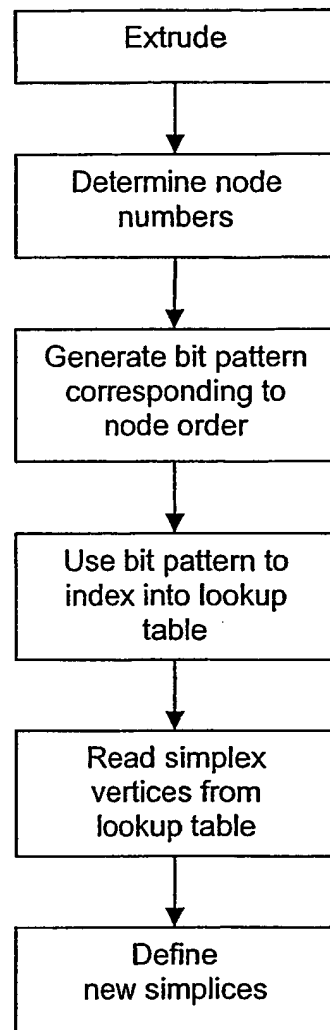
Fig. 7
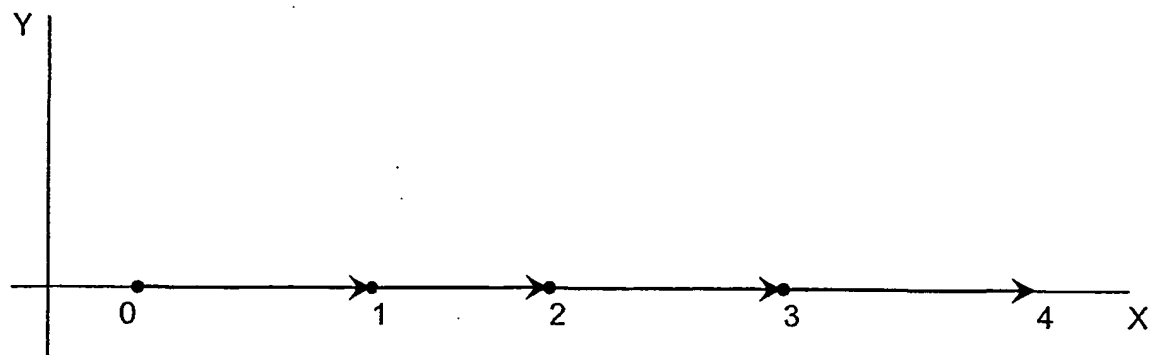
Fig. 8a
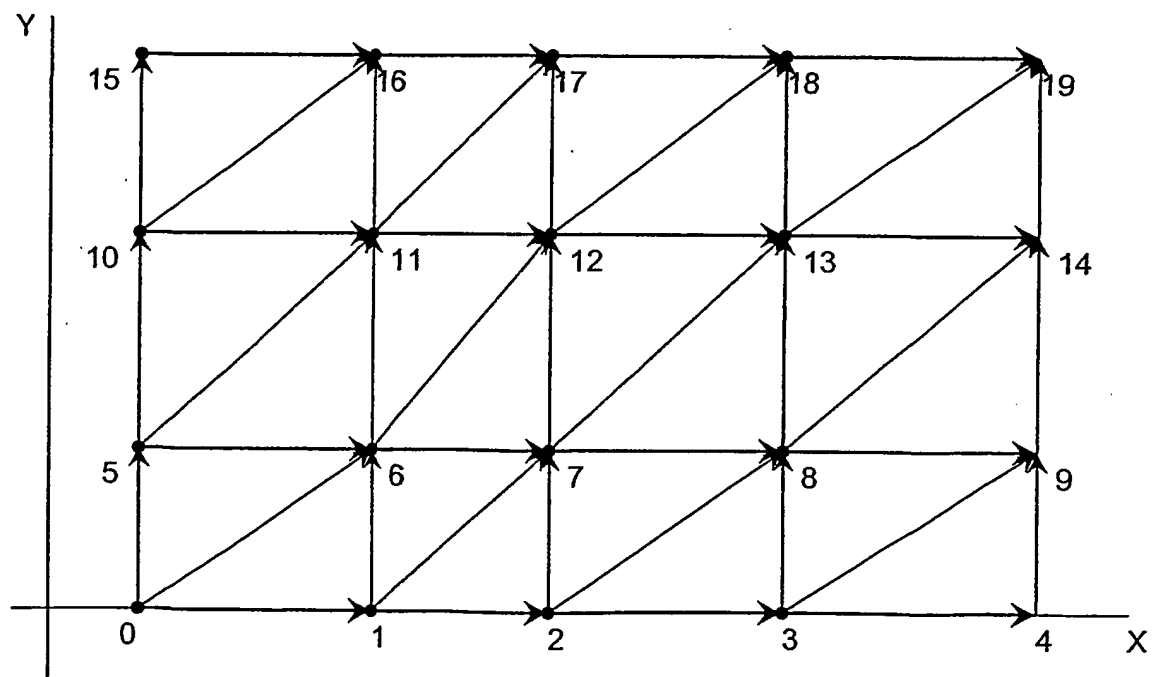
Fig. 8b
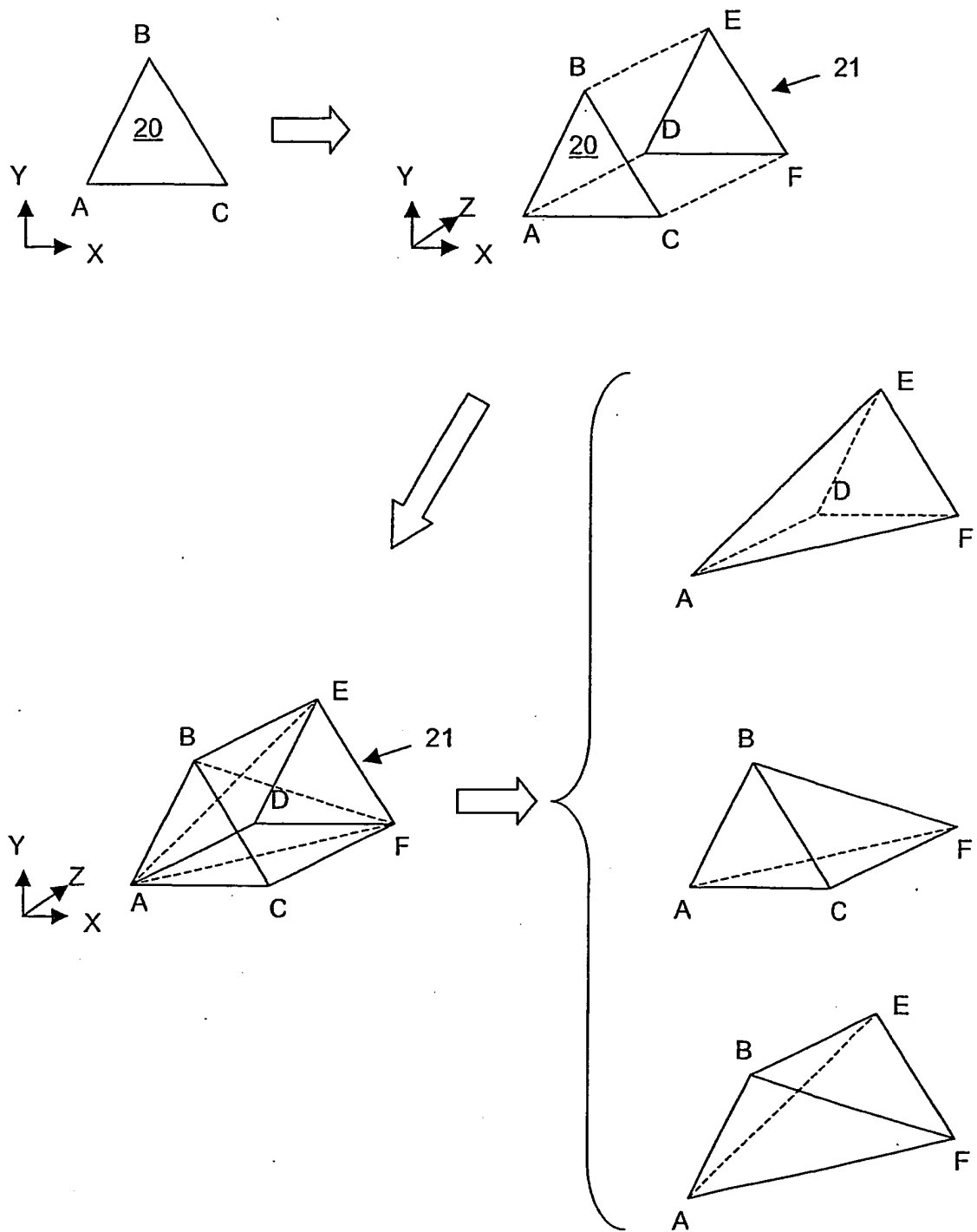
Fig. 9

INTERNATIONAL SEARCH REPORT

A. CLASSIFICATION OF SUBJECT MATTER
IPC 7   G06F17/50   G01V11/00

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC 7   G01V   G06F   G06T

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practical, search terms used)

EPO-Internal, INSPEC, COMPENDEX

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category°	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	US 6 052 520 A (WATTS III JAMES W) 18 April 2000 (2000-04-18)	1,2,14, 15
Y	column 1, line 16 -column 2, line 54 ·column 7, line 33 -column 8, line 47	4,17
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	-/--	

[X] Further documents are listed in the continuation of box C.   [X] Patent family members are listed in annex.

° Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance

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"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)

"O" document referring to an oral disclosure, use, exhibition or other means

"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention

"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone

"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art

"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
23 April 2002	0 6. 05. 02

Name and mailing address of the ISA	Authorized officer
European Patent Office, P.B. 5818 Patentlaan 2 NL – 2280 HV Rijswijk Tel. (+31–70) 340–2040, Tx. 31 651 epo nl, Fax: (+31–70) 340–3016	Lorne, B

Form PCT/ISA/210 (second sheet) (July 1992)

C.(Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication,where appropriate, of the relevant passages	Relevant to claim No.
A	PERROCHET P: "Finite hyperelements: a 4D geometrical framework using covariant bases and metric tensors" COMMUNICATIONS IN NUMERICAL METHODS IN ENGINEERING, JUNE 1995, UK, vol. 11, no. 6, pages 525-534, XP008000072 ISSN: 0748-8025 page 531, paragraph 2 -page 532	1,14
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A	US 5 617 322 A (YOKOTA IKUHIRO) 1 April 1997 (1997-04-01) column 2, line 60 -column 3, line 10; figure 21	27,31
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